Focusing of Eruptions by Fracture Wall Erosion

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Abstract. Lithospheric flexural stresses beneath volcanic loads are horizontally strongly compressive towards the top of the lithosphere. Thus, while magma transport through the brittle lithosphere occurs via fractures, the fracture paths under the volcanic center are predicted by stress trajectories to be horizontal and thus unable to supply melt to the volcanic edifice where eruptions are observed. Moreover, the magnitude of the compressive stresses under large loads would close down any vertical magma paths. Both problems may be resolved by additional stresses due to melting or thermomechanical erosion of fracture walls developing over the life-span of the volcano. Fractures form and close frequently in the seismogenic zone of the lithosphere, with each fracture eroding away a small amount of material. The total amount of material removed makes the stress field more tensile, thereby facilitating the long-lived and vertically oriented magma pathways necessary to build discrete volcanic structures.

Introduction

A first order observation of volcanic shield formation is that magma pathways through the lithosphere (including the oceanic crust) are highly localized; horizontal distribution along rift zones does not occur until the magma reaches the volcanic edifice [*Ryan*, 1988]. Continuous use or re-use of the same localized magma pathways throughout the entire evolution of the volcano ($\approx 10^6$ years [*Guillou et al.*, 1997]) results in the typical geometry of discrete shields instead of a continuous ridge along the hotspot path. Magma transport through the lithosphere most likely occurs via hydraulic fracturing [*Turcotte*, 1982], while interaction with flexural stresses due to the volcanic loads results in the characteristic spacing of volcanoes [*Ten Brink*, 1991; *Hieronymus and Bercovici*, 1999].

Observations of gravity and vertical seafloor displacement underneath and around volcanic seamounts and islands show that these features are flexurally supported by the lithosphere [*Watts*, 1978]. Beneath the volcanic load, horizontal flexural stresses are compressive near the surface and tensile towards the base of the lithosphere. Underneath a load the size of Mauna Loa, the magnitude of the maximum stress perturbations (both compressive and tensile) is of the order of 1 GPa. A continuous fracture or conduit through the lithosphere (from base to top) [*Aki*, 1981] cannot remain open in such a compressive stress regime unless the magma is highly pressurized; however, such pressurization is not consistent with observations of lava fountaining which indicate low overpressures [*Maaløe*, 1998] caused by shallow exsolution of gases. Alternatively, magma transport may occur in isolated melt pockets, i.e., fractures that are pinched off at the trailing end [*Maaløe*, 1998; *Rubin*, 1995]; however, the buoyancy which drives such fractures upward is offset by the downward gradient of flexural stresses [*Rubin*, 1995]. Under large loads, this stress gradient dominates and tends to force the melt pockets downward.

The orientation of the stress field also poses a problem for magma transport via fractures. In a volcanically loaded homogeneous lithosphere, fracture paths — which, over long distances, must be perpendicular to the least compressive principal stress [*Rubin*, 1995] — would level out in the upper few kilometers of the lithosphere (Figure 1). The vertical fractures necessary to transport magma into the volcanic edifice cannot form in such a stress field. A model based exclusively on fractures interacting with flexural stresses thus
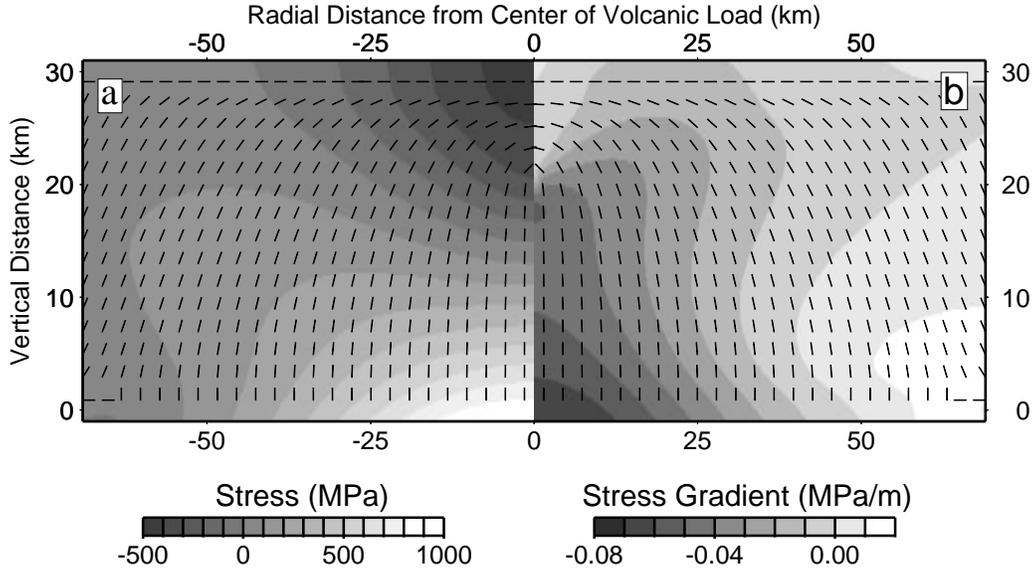
Figure 1. Cross-section through lithosphere indicating stress perturbation due to flexure and compression by a volcanic load. Ticks are perpendicular to the least compressive principal stress (i.e. in the direction of fracture propagation). Most tensile stress under volcano is radial near the base and vertical near the surface. (a) Shading indicates magnitude of maximum principal stress. Negative (compressive) stresses indicate region where fractures cannot remain open. (b) Shading shows gradient of most tensile principal stress along the fracture direction minus the buoyancy of melt in the lithosphere, which is $\Delta\rho g \approx 0.005$ MPa m^{-1}. Negative values indicate that underneath the volcanic edifice, melt pockets would be forced downward and radially outward by the stresses. Stresses were calculated using three-dimensional Maxwell stress functions *Leipholz* [1974] and spectral methods with 512 modes over 1000 km in both x and y; the figure zooms in on the central portion. The volcanic load is a Gaussian of amplitude 350 MPa and half width 39 km. The lithosphere rests on a liquid substratum of density 3000 kg/m^3. Elastic lithospheric thickness is 30 km *Watts* [1978] with constant elastic parameters.

cannot easily explain the observed volcanism and the formation of large shield volcanoes.

The results of a simple, horizontally two-dimensional model [*Hieronymus and Bercovici*, 1999] suggest that melt-back and thermomechanical erosion of the walls of feeder dikes may provide a dynamical feedback which counteracts the flexural stresses and keeps the magma channels stationary. Melt-back of fracture walls is possible if the fracture is wide enough initially and the magma flux is sufficient [*Lister and Dellar*, 1996; *Bruce and Huppert*, 1990].

In this paper, we show that erosion of only a small amount of material suffices to overcome the compressive flexural stresses locally, thus making the lithosphere beneath the volcanic center the preferred pathway for subsequent fractures to the surface.

The Model

We assume that the effect of magma transport (after opening and closing of fractures) is on average a removal of ma-

terial via melt-back and erosion of country rock. If a large number of temporary fractures form over the life span of a volcano, then their net effect may be approximated in a statistical sense. Thus, we assume a Gaussian distribution of rock removal in both horizontal dimensions. As long as the net stress is compressive (due to lithostatic pressure), the fractures will reclose after passage of a magma pulse and the stress field may be calculated using continuum mechanics. Distributed removal of material has the same effect on the stress field as local thermal contraction, as both effect an isotropic volume change. Using this analogy, we may write for the stresses (following *Timoshenko and Goodier* [1970]),

$$\sigma_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2G\epsilon_{ij} - \left(\tfrac{2}{3}G + \lambda\right)\Theta\delta_{ij} \qquad (1)$$

where $\Theta(x, y, z)$ is the volume strain due to material removal, σ_{ij} and ϵ_{ij} (i, j=1 to 3) are the components of the stress and strain tensors, respectively, λ is the second Lamé constant, and G is the elastic shear modulus. Summation is implied over repeated indices, and we adopt the sign convention wherein tensile stresses are positive. Combined with the compatibility equations, the equations of dynamical equilib-

rium [*Timoshenko and Goodier*, 1970], appropriate boundary conditions, and the function $\Theta(x, y, z)$, the six independent components of (1) form a complete set. Solutions may be found by using a displacement potential Ψ [*Timoshenko and Goodier*, 1970] where

$$\epsilon_{ij} = \frac{\partial^2 \Psi}{\partial x_i \partial x_j} \qquad (2)$$

and Ψ is a solution of

$$\nabla^2 \Psi = \frac{1}{3} \left(\frac{2G + 3\lambda}{2G + \lambda} \right) \Theta. \qquad (3)$$

Material removal results in a negative volumetric strain which, for simplicity, we assume has a Gaussian spatial distribution of the form

$$\Theta = -\Theta_{\max} \exp \left(-\frac{x^2 + y^2}{r_v^2} \right) \qquad (4)$$

centered beneath the volcanic load at $x = y = 0$, where $\Theta_{\max} > 0$, and r_v is the half-width of the strain distribution. The stresses due to material removal are calculated using (1)–(3); these add linearly to the flexural stresses (shown in Figure 1) to yield the total stress field. Equation (3) is solved spectrally in the horizontal domain using the plane stress approximation, which is exact if Θ is independent of depth. The boundary conditions are zero stress on the top and bottom boundaries and on vertical boundaries far from the region of interest.

Results

Calculations of the net stress field (due to flexure plus mass removal) are made for a variety of maximum volumetric strains $\Theta_{\max}$; the half-width, however, is fixed at $r_v = 5$ km which is estimated from the size of the seismically active region beneath Kilauea [*Ryan*, 1988]. A sample solution is shown in Figure 2 which depicts stresses resulting from material removal with a maximum volumetric strain of $\Theta_{\max} = 0.016$ (just large enough to cause net horizontal tensile stress perturbation; see below in discussion of Figure 4). This example demonstrates that a relatively minor amount of material removal results in a significant tensile stress; at $\Theta_{\max} = 0.016$, the resulting stress change is on the order of 1 GPa. The volume of material removed is $2\pi\Theta_{\max} H \int_0^\infty \exp(-r^2/r_v^2) r \, dr = \pi\Theta_{\max} H r_v^2 \approx 101$ km^3, assuming a thermal lithospheric thickness of $H = 80$ km, as is appropriate for the example of the Hawaiian Islands [*Doin and Fleitout*, 1996]. This volume compares to a total volcanic edifice volume of Mauna Loa of 42,500 km^3, indicating about 0.24 % of lithospheric contamination in the volcanic rock. This value is at the lower end of estimates
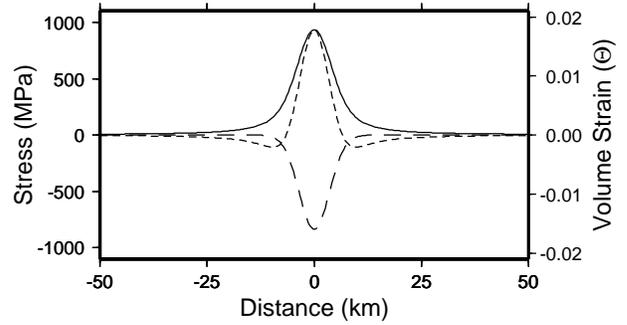

Figure 2. Cross-section of volume strain due to material removal by erosion (long dashes), and resulting radial (solid line) and azimuthal (short dashes) stresses. Vertical stress is zero and all functions plotted are constant with depth.

of lithospheric contamination in hotspot lavas based on osmium isotopes [*Widom et al.*, 1999].

Figure 3 shows the sum of the flexural stress field (Figure 1) and the sample stress field due to volume removal (Figure 2). Perhaps the most important effect of material removal is to change the direction of the least compressive principal stress. Since the stresses due to volume removal are tensile only in the horizontal direction, the effect on the total stress field is to make the fracture paths nearly vertical directly beneath the volcanic summit. Moreover, the magnitude of the least compressive stress is now tensile (relative to lithostatic) for a number of paths that focus towards the volcanic center. With the choice of Θ which is independent of depth, the gradient of the most tensile stress along the fracture path relative to melt buoyancy (Figure 3b) still does not allow isolated melt pockets to reach the volcanic centre. In fact, the near-surface fracture paths are vertical now, a direction in which the stress gradient is more prohibitive to magma transport toward the volcano than without erosional stresses.

The general dependence of total stress and inferred fracture trajectory on mass removal (in terms of both volumetric strain $\Theta_{\max}$ and magmatic contamination) is shown in Figure 4. Horizontal stresses remain compressive until the volume strain reaches $\Theta_{\max} \approx 0.015$ (Figure 4a), after which horizontal stresses become tensile. The orientation of the fracture trajectories underneath the center of the volcano changes abruptly from horizontal to vertical at $\Theta_{\max} \approx 0.009$. Note that the switch in trajectory (Figure 4b) occurs before the horizontal stress perturbations become tensile (Figure 4a); this disparity arises from the fact that, while the horizontal stress perturbations at $\Theta_{\max} \approx 0.009$ are still compressive, they have become less compressive than the vertical stress due to the weight of the volcano, and thus the stress/fracture trajectory switches from horizontal to vertical.
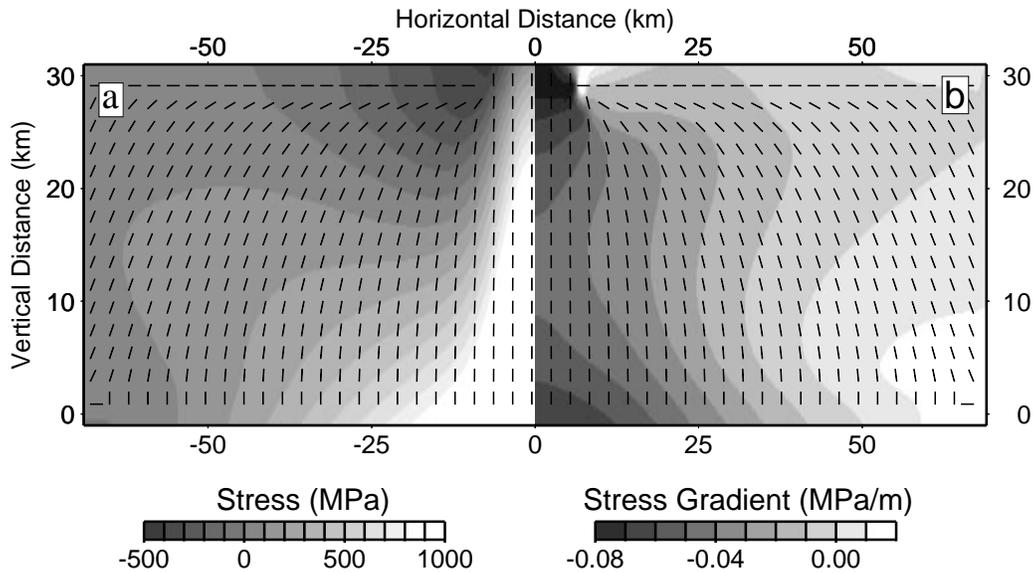
Figure 3. Lithospheric stress field of Figure 1, added to stresses due to material removal of Figure 3. Ticks and shading have same meaning as in Figure 1.

Discussion

The persistence of magma channels underneath the centers of large volcanoes may be explained by the results illustrated in Figure 3. If magma fractures close and re-form, and if the net result of each fracture is the removal of some small amount of wall rock by melting or thermo-mechanical erosion, then the stress field may become much more tensile in the central region underneath the volcano. In such a weakly compressive or tensile stress regime, new fractures can form even with relatively low magma pressure, existing fractures are not forced shut by the flexural stress, and the fracture paths are indeed vertical as indicated by the direction of most compressive total stress. Upward propagation is possible as long as the fracture remains connected to a pressurized magma reservoir at the base of the lithosphere.

As demonstrated in Figure 3b, our simple model suggests that isolated melt pockets are unable to travel upward through the lithosphere because their buoyancy is insufficient to overcome the along-path gradient of the most tensile principal stress, which tends to force the fractures downward. The notion of isolated melt pockets ascending through the lithosphere [*Rubin*, 1995] results from consideration of buoyant fractures in a remote stress field that is lithostatic. [*Secor and Pollard*, 1975; *Weertman*, 1971]. Both dynamic pressure due to viscous flow of magma within the fracture [*Spence and Turcotte*, 1985], and the flexural stress field, which becomes increasingly tensile with depth, reduce the tendency of the fracture to close off at the base of the litho-

sphere. Isolated fractures may therefore not be the mechanism of magma transport underneath large mid-plate volcanoes. These theoretical considerations are supported by observations of the magnitude of deep harmonic tremors, which suggest that much of the magma transport occurs aseismically in channels which remain open over longer time scales [*Aki*, 1981]; the model assumption of distributed material removal is still valid if fractures exist on time scales of years or decades.

A further consequence of the tensile flexural stresses at the base of the lithosphere is that vertical magma-filled fractures entering the lithosphere from a magma reservoir kept at lithostatic pressure will be strongly overpressurized with respect to the surrounding rock. In such an environment, a fracture will not close if the magma supply is depleted, but will instead solidify in situ, resulting in addition of material to the lithosphere at depth. Intrusion and solidification of dikes is expected in a large horizontal region where the flexural stress field is tensile and where melt is available. This volume increase may have a significant effect both on the stress field and on the flexural deflection. The model introduced above, with erosion of lithospheric rock that is independent of depth, is probably an oversimplification; a more adequate model might add material at depth while removing rock at shallow levels. A complete treatment of the mechanics and thermodynamics of the individual fractures and the surrounding rock would be required to find the appropriate function $\Theta(x, y, z)$, which is beyond the scope of this paper.
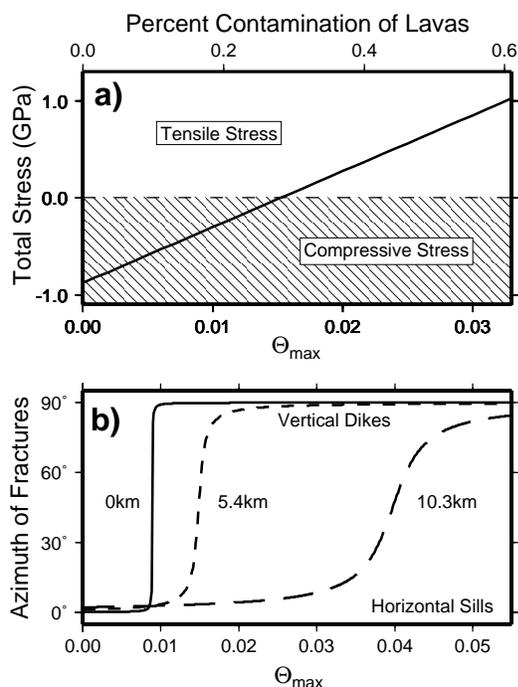
Figure 4. (a) Radial stress perturbation due to volcanic load, flexure and volume strain underneath volcanic center at top of lithosphere as a function of Θ_{max} (with r_v=5 km) and corresponding percent contamination of lavas (based on example of Mauna Loa). (b) Azimuth of most compressive total stress as a function of Θ_{max} at a depth of 2.5 km and a distance from the volcanic center of r=0 km, 5.4 km and 10.3 km.

The simple model does, however, demonstrate that erosion of wall rock may change the stress field significantly and may resolve the problems associated with maintaining magma pathways to the surface in the presence of strongly compressive stresses and horizontal stress trajectories beneath mid-plate volcanoes.

References

Aki, K., and R. Koyanagi, Deep volcanic tremor and magma ascent mechanism under Kilauea, Hawaii, *J. Geophys. Res., 86,* 7095-7109, 1981.

Bruce, P. M., and H. E. Huppert, Solidification and melting along dykes by the laminar flow of basaltic magma, in *Magma Transport and Storage,* edited by M. P. Ryan, pp. 87-101, John Wiley, New York, 1990.

Doin, M. P., and L. Fleitout, Thermal evolution of the oceanic lithosphere: an alternative view, *Earth Planet. Sci. Lett., 142,* 121-136, 1996.

Guillou, H., M. O. Garcia, and L. Turpin, Unspiked K-Ar dating of young volcanic rocks from Loihi and Pitcairn hot spot seamounts, *J. Volc. Geotherm. Res., 78,* 239-249, 1997.

Hieronymus, C. F., and D. Bercovici, Discrete and alternating hotspot islands formed by interaction of magma transport and lithospheric flexure, *Nature, 397,* 604-607, 1999.

Leipholz, H. *Theory of Elasticity,* 400 pp., Noordhoff International Pub., Leyden, Netherlands, 1974.

Lister, J. R., and P. J. Dellar, Solidification of pressure-driven flow in a finite rigid channel with applications to volcanic eruptions, *J. Fluid Mech., 323,* 267-283, 1996.

Maaløe, S., Shape of ascending feeder dikes, and ascent modes of magma, *J. Volc. Geotherm. Res., 81,* 207-214, 1998.

Rubin, A. M., Propagation of magma-filled cracks, *Annu. Rev. Earth Planet. Sci., 23,* 287-336, 1995.

Ryan, M. P., The mechanics and three-dimensional internal structure of active magmatic systems: Kilauea volcano, Hawaii, *J. Geophys. Res., 93,* 4213-4248, 1988.

Secor, D. T., and D. D. Pollard, On the stability of open hydraulic fractures in the Earth's crust, *Geophys. Res. Lett., 2,* 510-513, 1975.

Spence, D. A., and D. L. Turcotte, Magma-driven propagation of cracks, *J. Geophys. Res., 90,* 575-580, 1985.

Ten Brink, U., Volcano spacing and plate rigidity, *Geology, 19,* 397-400, 1991.

Timoshenko, S. P., and J. N. Goodier *Theory of Elasticity,* 567 pp., McGraw-Hill, New York, 1970.

Turcotte, D. L., Magma migration, *Ann. Rev. Earth Planet. Sci., 10,* 397-408, 1982.

Watts, A. B., An analysis of isostasy in the world's oceans: 1. Hawaiian-Emperor seamount chain, *J. Geophys. Res., 83,* 5989-6004, 1978.

Weertman, J., Theory of water-filled crevasses in glaciers applied to vertical magma transport beneath oceanic ridges, *J. Geophys. Res., 76,* 1171-1183, 1971.

Widom, E., K. A. Hoernle, S. B. Shirey, and H.-U. Schmincke, Os isotope systematics in the Canary Islands and Madeira: lithospheric contamination and mantle plume signatures, *J. Pet., 40,* 279-296, 1999.

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