GEOPHYSICAL RESEARCH LETTERS, VOL. 19, NO. 17, PAGES 1791-1794, SEPTEMBER 4, 1992

WAVE DYNAMICS IN MANTLE PLUME HEADS AND HOTSPOT SWELLS

David Bercovici

Department of Geology and Geophysics, University of Hawaii, Honolulu

Abstract. Laboratory experiments with thermal plumes in fluid with temperature-dependent viscosity suggest that wave-like instabilities can form in the horizontally flowing, disk-shaped head of the plume. The waves propagate radially outward from the axis of the plume and appear to be most intense in a finite band near the perimeter of the plume head. A simple theoretical model shows that interfacial waves in a highly viscous fluid may occur if the plumehead is comprised of temperature-dependent-viscosity fluid that cools as it flows between two boundaries. The model suggests that the waves arise as an oscillatory instability and that wave formation is most robust in the colder regions of the plume-head, as indicated by the experiments. The theory also predicts that the instability will only occur above some critical plume-head flow velocity, and that mantle plume conditions are generally supercritical.

Introduction

Thermal plumes in the Earth's mantle display significant time-dependent behavior. The existence of discrete islands in hotspot tracks (e.g., the Emperor-Hawaiian island chain) is evidence of the apparently pulsating nature of mantle plumes. Mechanisms to explain time-dependent behavior have primarily focussed on the dynamics of the plume conduits. Pulsating behavior has been attributed to the tilting and breakup of a plume conduit under large scale mantle shear flow (Skilbeck and Whitehead, 1978) or to solitary wave propagation along the plume conduit (Scott et al., 1986; Olson and Christensen, 1986). Here we present the results of laboratory experiments and theory which suggest that oscillatory behavior can also occur in mantle plume heads in the form of horizontally propagating waves. These waves possibly result from an oscillatory instability which can occur in the flow of cooling temperature-dependent-viscosity fluid. Wave propagation and oscillatory behavior in plume heads may influence hotspot temporal variability and the spatial structure of swells. In this note, we report the laboratory observation of the plume-head waves and present a simple theory to illustrate a possible mechanism for the waves.

Laboratory Experiments

The laboratory experiment involves heating "Karo" brand corn syrup to 70^oC in an isothermal bath and feeding the hot syrup into a glass tank ($50cm \times 65cm$ base; $25cm$ height) of room temperature (25^oC) corn syrup through a plastic pipe with an inner diameter of $1.5cm$ and an outer diameter of $2.0cm$. The depth of fluid in the tank is $z_f = 17cm$. The bath is placed approximately a meter above the tank. Hot syrup from the bath is fed at a constant rate into the pipe; a hydraulic head forms in the pipe which forces out the corn syrup. During the experiments the level of the head is, on average, $z_h = 110cm$ above the bottom (inside) of the tank. At the end of the pipe is a narrow nozzle through which the hot syrup flows upward to form a plume. The nozzle has a radius of $a = 0.15cm$ and a length of $z_n = 3cm$. The rising plume impinges on an overlying room-temperature glass plate and spreads outwards into a disk-shaped plume head. The distance between the tip of the nozzle and the glass plate is $1.25cm$. Once steady state is acheived, the plume head essentially maintains a constant thickness of $H_o = 0.5cm$. The level of fluid in the tank is maintained by syphoning off syrup from the bottom of the tank. Experimental observations are made using a shadowgraph technique. Either thermal anomalies or undulations in a fluid-fluid interface (which act as sequential concave and convex lenses) appear as bright and dark zones on the shadowgraph.

The dynamic viscosity of room temperature syrup is $\eta_c = 45P$ (Helfrich and Whitehead, 1990), and an increase in temperature to $70°C$ yields up to a factor of 100 decrease in viscosity based on typical viscosity laws for corn syrup (e.g., Weinstein and Christensen, 1991). The viscosity of fluid leaving the nozzle η_h may be estimated by comparing the observed volume flux through the pipe $Q_o \approx 1.7cm^3/s$ with the theoretical volume flux (assuming Poiseuille flow through the nozzle) $\frac{\pi a^4 \rho g(z_h - z_f + z_n)}{8\eta_h z_n}$ where $\rho = 1.42g/cm^3$ is the density of the corn syrup (Helfrich and Whitehead, 1990). This comparison indicates that η_h is only a factor of 10 less than η_c, possibly because of cooling in the pipe due to imperfect insulation. Vertically averaged horizontal velocity in the plume head is $\bar{U}_o \approx \frac{Q_o}{2\pi r H_o}$ (where r is the radial distance in the plume head from the plume axis), which yields Reynolds numbers Re of $\rho \bar{U}_o H_o/\eta_h \approx 0.1$ near the plume axis ($r = 1cm$), and $\rho \bar{U}_o H_o/\eta_c \approx 5 \times 10^{-4}$ near the plume-head perimeter ($r = 20cm$). While Re near the axis of the plume head is not $\ll 1$, the wave phenomenon, as discussed below, is most pronounced near the perimeter of the plume head where Re is unequivocally $\ll 1$. Thermal diffusivity κ of syrup is approximately $1.6 \times 10^{-3} cm^2/s$ (Weinstein and Christensen, 1991), thus the Peclet number Pe lies between 10 and 200. The occurrence of the waves in a low Re, high-to-moderate Pe flow lends credence to the applicability of these experiments to real mantle plumes.

Once the pancake-shaped plume head is formed, outwardly propagating, nearly concentric wave-like features can be observed (Figure 1). The features have wavelengths on the order of $1cm$ and propagation speeds on the order of $1cm/s$ near the plume axis and become imperceptibly slow at the perimeter of the plume head. The propagation speed of the features appears to be proportional to the velocity of the radial flow, thus the features may be at

Paper number 92GL01703
0094-8534/92/92GL-01703$03.00

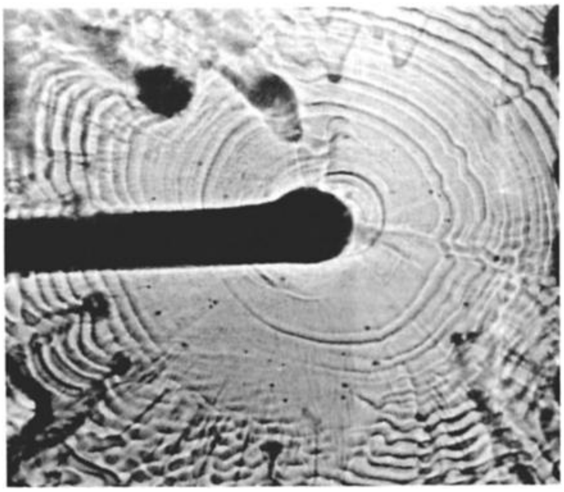

Fig. 1. Plan view shadowgraph of the plume head. Dark and light nearly concentric circles are the waves. The thick straight shadow is from the pipe feeding the plume; the width of the pipe (with attached metal spine) is 2.5*cm*, for scale. The small dark circular regions are areas in the plume head that have become cold and started to sink (this is also due to slight loss of water from the syrup while being heated in the isothermal bath). The thick, dark, irregular lines on the left and right are due to syrup overflowing onto the overlying glass plate from imperfect syphoning.

least partially advected by the radial flow. The features also become strongly pronounced in a finite band near the perimeter of the plume head. We infer their appearance in the shadowgraph to be due to undulations in the thickness of the plume head, i.e., waves on the fluid-fluid interface between the plume head and underlying colder syrup. (Surface tension effects, e.g., capillary waves, are not a consideration because the two fluids are completely miscible.) It is doubtful that the nearly axisymmetric wave-like features are thermal anomalies from small-scale convection. Given the experimental parameters, the thin plume head is probably not convectively unstable. Moreover, convection in the presence of horizontal flow tends to assume a pattern of rolls aligned in the direction of flow if the flow is relatively fast (e.g. Richter and Parsons, 1975), or a three-dimensional spoke pattern if the flow is slow and the upper surface is rigid (as in these experiments) (Griffiths and Campbell, 1991).

The plume-head waves appear consistently in seven experimental trials, though they are most pronounced when initiated by squeezing the conduit once or twice at the start of the experiment. Once started, the waves persist for the duration of the experiment. The occurrence of the waves is typically accompanied by oscillations in the level of the hydraulic head in the pipe with frequencies on the order of $1Hz$ and greater. Whether the waves cause the pressure oscillations or vice-versa is not known, and it is possible that the fluid in the pipe could undergo oscillatory behavior (due to cooling in the pipe; see Whitehead and Helfrich, 1991) which then excites waves in the plume head. However, the waves in the plume head are typically faint near the axis of the plume and grow toward the perimeter of the head; it is doubtful that oscillations in the pipe could cause such behavior. A dynamic mechanism is necessary to allow the waves to propagate undamped across a viscous plume head, let alone experience growth.

Theory

Our working hypothesis is that the observed waves are intrinsic to thermal plumes in fluid with temperature-dependent viscosity. Isoviscous theory can predict no more than rapidly decaying waves, and experiments with isothermal (i.e., chemical) plumes have not reported similar wave-like features in the plume head (e.g., Griffiths and Campbell, 1991). Alternatively, Whitehead and Helfrich [1991] demonstrated that the flow of a cooling, temperature-dependent-viscosity fluid can undergo oscillations and fingering. However, their theoretical model was most applicable to moderate and low Reynolds number flows under quasi-static thermal equilibrium. This is not completely appropriate for viscous plumes. Here, we present a simple theory to illustrate how a cooling temperature-dependent-viscosity fluid flowing between two parallel boundaries, in which one boundary is deformable, can experience an oscillatory wave-like instability similar to the observed plume-head waves. For simplicity, we employ a model that differs slightly from the experiment to facilitate a more tractable analysis. In the theoretical model, fluid flowing between the boundaries is heated and cooled uniformly (at the boundaries and/or internally) in the horizontal direction such that the steady state temperature is constant in this direction. In contrast, heat in the experiment is supplied from the source of fluid and advected toward the perimeter of the plume head. The theory and experiment thus differ in the mode by which heat is supplied. The model, however, serves to illustrate the essential physics.

Although the plume head is nearly an axisymmetric disk, for simplicity we model it as an infinitely long two-dimensional Cartesian channel of thickness H. The channel is bounded above by a no-slip, rigid boundary and below by a deformable boundary underlain by a denser, much more viscous fluid, such that the boundary is assumed no-slip also. (The lower boundary condition is a significant simplication as it requires the underlying fluid to have a much higher viscosity than the cold plume-head fluid.) Using a friction factor approach (Bird et al., 1960), the vertically averaged equation of motion for incompressible Stokes flow in the x direction (i.e., parallel to the boundaries of the channel) is

$$\frac{12\eta(\bar{\Theta})}{H^2}\bar{U} = -\frac{\partial P}{\partial x} \quad (1)$$

where $\bar{U}$ is the vertical average of velocity in the x direction, and the nonhydrostatic pressure P is assumed independent of z. The temperature-dependent viscosity η is, for simplicity, prescribed by

$$\eta(\bar{\Theta}) = \frac{\eta_h \eta_c}{\eta_h + \eta' \bar{\Theta}} \quad . \quad (2)$$

$\bar{\Theta}$ is the vertically averaged dimensionless temperature where $0 \leq \bar{\Theta} \leq 1$. η_h and η_c are the minimum and maximum viscosities, respectively, and $\eta' = \eta_c - \eta_h$. This inverse dependence of viscosity on temperature roughly approximates the rheology of highly viscous materials (i.e., viscosity is more sensitive to thermal fluctuations at colder temperatures). By conservation of mass,

$$\frac{\partial H}{\partial t}+\frac{\partial(\bar{U}H)}{\partial x}=0 \quad . \tag{3}$$

We also employ a one-dimensional advection-diffusion equation for $\bar{\Theta}$:

$$\frac{\partial\bar{\Theta}}{\partial t}+\bar{U}\frac{\partial\bar{\Theta}}{\partial x}=-\frac{\kappa}{H^2}\bar{\Theta}+\epsilon \tag{4}$$

where diffusion is represented by a heat transfer coefficient for diffusion accross the channel (geometrical factors being absorbed into the thermal diffusivity κ; see also Whitehead and Helfrich, 1991) and ϵ represents internal heating or cooling. Dependent variables are linearized around steady state channel flow such that $\bar{U}=\bar{U}_o+\bar{u}$, $\bar{\Theta}=\bar{\Theta}_o+\bar{\theta}$, $H=H_o+h$ and $P=P_o(x)+\Delta\rho g h$ (where $\Delta\rho$ is the density contrast between fluid in the channel and the underlying fluid, and g is gravitational acceleration). P_o is a pressure whose horizontal gradient drives fluid with average temperature $\bar{\Theta}_o$ through a uniform channel of width H_o at a steady velocity $\bar{U}_o=-\frac{H_o^2\eta_o}{12\eta_h\eta_c}\frac{dP_o}{dx}$, where $\eta_o=\eta_h+\eta'\bar{\Theta}_o$. Lower case variables represent infinitesimal perturbations. The contribution of normal viscous stresses to P is neglected since these stresses scale as $(H_o/R)^n$ ($n\geq 2$) where R, the horizontal length scale of the plume head, is $\gg H_o$. Finally, we nondimensionalize t by H_o^2/κ, $\bar{U}_o$ and $\bar{u}$ by $\kappa R/H_o^2$, h by H_o, and x by R, where we choose $R=\sqrt{\Delta\rho g H_o^5/(12\eta_h\kappa)}$. With linearization and nondimensionalization, solving for $\bar{u}$ via (1) and (2), and given that $\bar{\Theta}_o$ is constant in the theory and satisfies steady-state diffusion, equations (3) and (4) become

$$\frac{\partial h}{\partial t}+3\bar{U}_o\frac{\partial h}{\partial x}-\frac{\eta_o}{\eta_c}\frac{\partial^2 h}{\partial x^2}+\bar{U}_o\frac{\eta'}{\eta_o}\frac{\partial\bar{\theta}}{\partial x}=0 \tag{5}$$

$$\frac{\partial\bar{\theta}}{\partial t}+\bar{U}_o\frac{\partial\bar{\theta}}{\partial x}+\bar{\theta}-2\bar{\Theta}_o h=0 \quad . \tag{6}$$

Because of the unspecified heat source/sink ϵ, $\bar{\Theta}_o$ can assume any value between 0 and 1, and we henceforth treat it as a free parameter. Combining (5) and (6) and assuming that h and $\bar{\theta}\sim e^{ikx+\sigma t}$ we obtain the dispersion relation

$$\begin{aligned}\sigma^2+\left(\tfrac{\eta_o}{\eta_c}k^2+1+i4k\bar{U}_o\right)\sigma-\left(3\bar{U}_o^2-\tfrac{\eta_o}{\eta_c}\right)k^2\\ +ik\bar{U}_o\left(3+\tfrac{2\eta'}{\eta_o}\bar{\Theta}_o+k^2\tfrac{\eta_o}{\eta_c}\right)=0 \quad .\end{aligned} \tag{7}$$

Two complex roots for σ arise from (7), only one of which (the "+"-root) yields instabilities (i.e., can have $Re(\sigma)>0$). Figure 2 (inset) shows $Re(\sigma)$ versus k for the positive-growth root with various values of $\bar{U}_o$. Perturbations can have a positive growth rate which maximizes at a single wavenumber k implying that there is a preferred wavelength for instability. Though not shown, the frequency $Im(\sigma)$ is nearly a linear function of k; in the region of k where $Re(\sigma)$ is maximum, $Im(\sigma)\approx -ikc$ where $3\bar{U}_o\leq c\leq 5\bar{U}_o$. The wave thus travels to the right (i.e., in the direction of the flow) with phase speed proportional to the flow speed, as implied by the experiments.

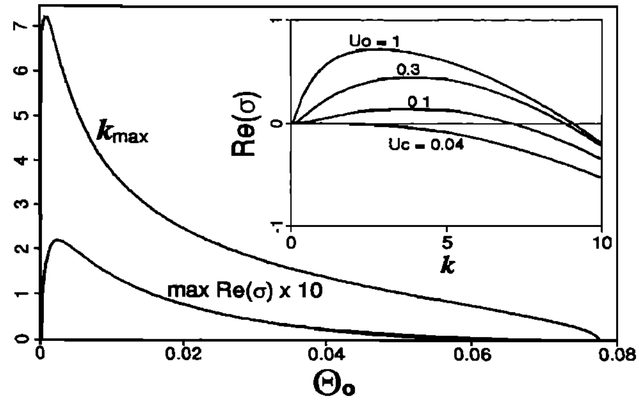

Fig. 2. Inset: dimensionless growth rate $Re(\sigma)$ versus wavenumber k (for dimensionless temperature $\bar{\Theta}_o=10^{-2}$ and various dimensionless horizontal velocities $\bar{U}_o$) for the theoretical model. Outer figure: maximum growth rate $Re(\sigma)$ and corresponding wavenumber k_{max} (i.e., the k at which $Re(\sigma)$ is maximum) versus temperature $\bar{\Theta}_o$ for $\bar{U}_o=0.1$. The viscosity ratio is $\eta_c/\eta_h=1000$ in both figures.

We can use the fact that $Re(\sigma)$ is greater than 0 in a finite band of k (i.e., between $k=0$ and some $k>0$) to obtain a critical $\bar{U}_o$ for the onset of instability. From (7), the boundaries of the band (i.e, the k at which $Re(\sigma)=0$) are at $k=0$ and

$$k=\sqrt{\frac{\eta_c}{\eta_o}}\left[\frac{1+\frac{2\eta'}{\eta_o}\bar{\Theta}_o-\sqrt{1+\frac{\eta_o}{\eta_c}\frac{1}{\bar{U}_o^2}}}{1+\sqrt{1+\frac{\eta_o}{\eta_c}\frac{1}{\bar{U}_o^2}}}\right]^{1/2} \quad . \tag{8}$$

The band has finite width only if $\bar{U}_o>\bar{U}_c=\sqrt{\frac{\eta_o}{\eta_c}}\left[\left(1+\frac{2\eta'}{\eta_o}\bar{\Theta}_o\right)^2-1\right]^{-1/2}$; for $\bar{U}_o\leq\bar{U}_c$, $Re(\sigma)$ is everywhere ≤ 0 (see Figure 2). Thus $\bar{U}_c$ is the critical velocity above which instability can occur. The instability is entirely precluded when $\bar{\Theta}_o=0$ or $\eta'=0$ (i.e., $\eta_c/\eta_h=1$); thus, the instability cannot occur if no heat is transferred (i.e., all the fluid is at the coldest temperature) or if viscosity is constant. For a given η_c/η_h, the minimum value of $\bar{U}_c$ occurs when $\bar{\Theta}_o=\bar{\Theta}_m=\left[(\sqrt{3}-1)(\eta_c/\eta_h-1)\right]^{-1}$. Thus, we can expect that for a given supercritical $\bar{U}_o$ the fluid is most unstable when $\bar{\Theta}_o=\bar{\Theta}_m$, which for $\eta_c/\eta_h\gg 1$ is near the minimum temperature of $\bar{\Theta}_o=0$.

Figure 2 also shows the maximum $Re(\sigma)$ and corresponding k versus $\bar{\Theta}_o$ for specific $\bar{U}_o$ and η_c/η_h. In this case, the instability is restricted to the temperature range $0\leq\bar{\Theta}_o\leq 0.08$, and the growth rate and corresponding k peak close to $\bar{\Theta}_o=\bar{\Theta}_m\ll 1$. Thus, the fluid is most unstable (or only unstable) at colder temperatures, and the least stable wave has the shortest wavelength. Although the theory is not rigorously applicable to the experiment, this prediction appears to be born out in that the laboratory waves are most pronounced in (or restricted to) the outer, colder regions of the plume head. (Either larger amplitude or smaller wavelength undulations will cause a more distinct appearance on the shadowgraph.)

The critical velocity may be used to estimate whether such an instability can occur in a real mantle plume head. For $\eta_c/\eta_h = 1000$ (a typical value for a mantle plume) and temperatures in the range $\bar{\Theta}_m \leq \bar{\Theta}_o \leq 1$ (assuming that instabilities are largely precluded for $\bar{\Theta}_o < \bar{\Theta}_m$), critical velocities are in the range $0.03 \leq \bar{U}_c \leq 0.35$. Redimensionalizing velocity using the mantle-type values $\Delta\rho = 35kg/m^3$ (i.e., a 1% average density anomaly for plume material), $g = 10m/s^2$, $H_o = 100km$, $\eta_h = 10^{18}Pa\ s$ and $\kappa = 10^{-6}m^2/s$, we find that the dimensional critical velocity is in the range $0.2cm/yr \leq \bar{U}_c \leq 1.9cm/yr$, which is less than expected mantle plume velocities. Thus, mantle conditions are likely to be supercritical for this plume-wave instability, especially in the colder regions of the plume head.

The mechanism for oscillation (or wave propagation, only part of which is due to advection) may be understood qualitatively by imagining an area of the channel thinning slightly under some pressure perturbation. Because of enhanced diffusion, that region becomes colder and thus forms a plug due to the consequent increase in viscosity. Pressure builds up and the channel thickens behind the plug, causing diffusion to diminish and thus the fluid there heats up. The pressure eventually builds enough to push through the cold plug, yet because of thermal inertia (i.e., temperature anomalies do not diffuse away instantaneously) fluid entering the narrowed region is hotter than both the cold plug fluid and the original steady state fluid. (In the model of Whitehead and Helfrich (1991), fluid overshoots in a similar manner because of momentum inertia.) This leads to an excessive drop in flow resistance in the narrowed region, causing the high pressure region behind it to over-deflate, subsequently cooling and allowing the process to begin again.

The mechanism for the instability can be understood by considering how perturbations obtain energy on which to grow. Taking the product of $\bar{\theta}$ with (6) and integrating over one full wavelength (or to homogeneous end boundaries) in x, leads to an equation for the growth of energy in the instability:

$$\frac{d}{dt}\int \bar{\theta}^2 dx = -2\int \bar{\theta}^2 dx + 4\bar{\Theta}_o \int h\bar{\theta} dx \quad . \qquad (9)$$

The first term on the right side is necessarily an energy sink while the second term can be a source or sink. Since the second term is proportional to the correlation between h and $\bar{\theta}$, perturbations can only grow if h and $\bar{\theta}$ are less than 90° out of phase. This occurs because, when a wave-like temperature perturbation exists, a volume of fluid lying between relatively hot and cold regions will, for example, inflate if the cold region is downstream (since the cold region forms a plug). However, since flow resistance decreases with an increase in either temperature or channel width, the inflating volume fills up faster on the side that is hotter and wider, causing h and $\bar{\theta}$ to be less than 90° out of phase. Thus there is a net positive (negative) h over the hotter (colder) regions, causing thermal diffusion to decrease (increase), and thus an added accumulation (loss) of heat in the hotter (colder) regions, thereby leading to instability.

Summary and Conclusions

Laboratory experiments with thermal plumes in fluids with temperature-dependent viscosity suggest that outwardly propagating waves form in the horizontally spreading, disk-shaped head of the plume. These plume-head waves are apparently self-sustaining, which a theoretical model suggests is because the waves form as an oscillatory instability (i.e., the waves can continuously grow from small disturbances). The theory also indicates that the instability can occur at actual mantle-plume conditions. Both the experiment and theory indicate that the waves are most pronounced in the colder regions of the plume head. Thus, if the waves (or wave-like structure) are to be observed in an actual hotspot swell, they might be most noticeable near the flanks of the swell.

Acknowledgments. Thanks are due to John C. Lynch for assistance with the laboratory experiments, Karl R. Helfrich for invaluable discussions and advice, and Peter Olson and two anonymous reviewers for many helpful comments. This research was supported by a grant from the University Research Council of the University of Hawaii.

References

Bird, R. B., W. E. Stewart and E. N. Lightfoot, *Transport Phenomena*, Chapter 6, John Wiley, 1960.

Griffiths, R. W. and I. H. Campbell, Interaction of mantle plume heads with the Earth's surface and onset of small-scale convection, *J. Geophys. Res., 96*, 18,295–18,310, 1991.

Helfrich, K. R. and J. A. Whitehead, Solitary waves on conduits of buoyant fluid in a more viscous fluid, *Geophys. Astrophys. Fluid Dyn., 51*, 35–52, 1990.

Olson, P. and U. Christensen, Solitary wave propagation in a fluid conduit within a viscous matrix, *J. Geophys. Res., 91*, 6367–6374, 1986.

Richter F. and B. Parsons, The interaction of two scales of convection in the mantle, *J. Geophys. Res., 80*, 2529-2541, 1975.

Scott, D. R., D. J. Stevenson and J. A. Whitehead, Observations of solitary waves in a viscously deformable pipe, *Nature, 319*, 759–761, 1986.

Skilbeck, J. N. and J. A. Whitehead, Formation of discrete islands in linear island chains, *Nature, 272*, 499-501, 1978.

Weinstein, S. A. and U. Christensen, Convection planforms in a fluid with a temperature dependent viscosity beneath a stress-free upper boundary, *Geophys. Res. Lett., 18*, 2035–2038, 1991.

Whitehead, J. A. and K. R. Helfrich, Instability of flow with temperature-dependent viscosity: A model of magma dynamics, *J. Geophys. Res., 96*, 4145–4155, 1991.

D. Bercovici, Department of Geology & Geophysics, School of Ocean & Earth Science & Technology, University of Hawaii, Honolulu, HI 96822.

(Received February 10, 1992;
revised June 12, 1992;
accepted July 20, 1992.)